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Almost Separation of Bias Precipitates in the Estimator of 'Inverse of Population Mean' with Known Coefficient of Variation

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SUMMARY

The paper deals with the problem of estimating 'inverse of population mean' when coefficient of variation is known. A funnel connected with a filter-paper to filter the bias precipitate appearing in the estimators of the inverse of population mean is defined.

Key words : Bias precipitates, Linear variety of estimators, Mean square error, Coefficient of variation, Normal parent.

Introduction and Notations

In various investigations, the coefficient of variation shows stability and its value may be known accurately. The use of coefficient of variation as a priori has been made at a great length in the estimation of mean by several authors including Searls [4], Khan [3], Govindarajulu and Sahai [2] Gleser and Healy [1], Singh [7] [8], among others. Sen and Gerig [5], Sen [6] and Upadhyaya and Singh [14] have used the population shape parameters such as coefficient of skewness and kurtosis as apriori in addition to coefficient of variation in estimating the population mean.

The problem of estimation of the inverse of population mean arises in many situations, for instance, in Econometrics and Biological sciences ; see Zellner [15]. The conventional estimator of the inverse of population mean is the 'inverse of sample mean'. Improvements over the conventional estimator have been made by Srivastava and Bhatnagar [13] and Singh [9] in the situations, where population variance is known and unknown. Singh et al [11] have also improvements over conventional estimator of inverse of population mean using a priori information on shape parameters of population such as coefficient of skewness and kurtosis in addition to coefficient of variation.

A method adopted by Singh and Singh [12] to filter the bias precipitates from the estimators of inverse of population mean by using a funnel associated with a filter-paper is given. The apparatus consists of a linear variety of estimators and linear constraints. It would be seen that the chemicals (statistical constants) used for bias separation depend on the shape parameters of population and coefficient of variation. However, in case of normal population the reactants

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(chemicals) used for bias filtration depend only on a simple apriori coefficient of variation.

For the sake of simplicity, assume that the population is infinite. Let $y_1, y_2, \ldots, y_n$ be random sample of size n drawn from a population with mean μ and variance σ^2. It is assumed that the population coefficient of variation is known.

Let $\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$ and $s^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{(n-1)}$ be respectively the unbiased estimators of population mean μ and variance σ^2. The parameters under investigation is the 'inverse of population mean' $\theta = \frac{1}{\mu}$, $(\mu \neq 0)$. The usual estimator of θ is $\hat{\theta}_1 \frac{1}{\bar{y}} (\bar{y} \neq 0)$. Let $\hat{c} = \frac{s}{\bar{y}}$ be the consistent estimate of coefficient of variation $c = \frac{\sigma}{\mu}$. Further let

$$\bar{y} = \mu(1 + \delta\bar{y}) \quad \text{and} \quad \hat{c} = c(1 + \delta\hat{c})$$

so that $E(\delta\bar{y}) = 0$, $E(\delta\bar{y}^2) = \frac{c^2}{n}$ and so the first degree approximation,

$$E(\delta\bar{c}) = -\frac{1}{8n}[\beta_2 - 1 + 4\sqrt{\beta_1}\, c - 8c^2] \tag{1.1}$$

$$E(\delta\bar{y}\,\delta\hat{c}) = -\frac{1}{2n}(2c^2 - \sqrt{\beta_1}\, c) \tag{1.2}$$

$$E(\delta\hat{c}^2) = \frac{1}{4n}(4c^2 - 4\sqrt{\beta_1}\, c + \beta_2 - 1) \tag{1.3}$$

where $c = (\frac{\sqrt{\mu_2}}{\mu}) = \frac{\sigma}{\mu}$, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$, $\beta_2 = \frac{\mu_4}{\mu_2^2}$ and μ_r, $(r = 2, 3, 4)$ is the r-th central moment.

2. *Linear Variety*

Suppose $\hat{\theta}_1 = \frac{1}{\bar{y}}$, $\hat{\theta}_2 = \frac{1}{\bar{y}}\left(\frac{\hat{c}}{c}\right)$ and $\hat{\theta}_3 = \frac{1}{\bar{y}}\left(\frac{c}{\hat{c}}\right)$ such that $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \in$ G where G denotes the set of all possible estimators for estimating the 'inverse of population mean' $\theta = \frac{1}{\mu}$. By definition, the set G will be linear variety if

$$\hat{\theta}_g = \sum_{i=1}^{3} g_i \, \hat{\theta}_i \tag{2.1}$$

for

$$\sum_{i=1}^{3} g_i = 1 \tag{2.2}$$

and $g_i \in R$

where g_i (i = 1, 2, 3) denote the amount of chemicals used for bias precipitates separation and R stands for the set of real numbers.

3. *Mean Square Error*

Expressing $\hat{\theta}_g$ in terms of $\delta\bar{y}$ and $\delta\hat{c}$, we have

$$= \delta\left[g_1 (1+\delta\bar{y})^{-1} + g_2(1+\delta\bar{y})^{-1} (1+\delta\hat{c}) + g_3 (1+\delta\bar{y})^{-1} (1+\delta\hat{c})^{-1}\right] \tag{3.1}$$

which may be expressed as

$$\hat{\theta}_g = \theta \, (1-\delta\bar{y}) + \theta (g_2 - g_3) \, \delta\hat{c} + 0(\delta^2) \tag{3.2}$$

Let us choose

$$g_2 - g_3 = g \qquad \text{(say, another constant)} \tag{3.3}$$

Then $\hat{\theta}_g = \theta (1-\delta\bar{y}) + \theta g \, \delta \hat{c} + 0(\delta^2)$

or

$$(\hat{\theta}_g - \theta) = -\theta \, \delta\bar{y} + \theta g \delta\hat{c} + 0(\delta^2) \tag{3.4}$$

Squaring both sides of (3.4) and retaining terms upto second powers of δ's we have

$$(\hat{\theta}_g - \theta)^2 = \theta^2\left[\delta\bar{y}^2 + g^2 \, \delta\hat{c}^2 - 2g \, \delta\bar{y} \, \delta\hat{c}\right] \tag{3.5}$$

Taking expectation of both sides of (3.5) and using $E(\delta\bar{y}^2) = \frac{c^2}{n}$, (1.1), (1.2) and (1.3) we get the mean square error of $\hat{\theta}_g$, to the first degree of approximation as

$$\text{MSE}(\hat{\theta}_g) = \left(\frac{\theta^2}{4n}\right)\left[g^2 (4c^2 - 4\sqrt{\beta_1}\, c + \beta_2 - 1) + 4g(2c^2 - \sqrt{\beta_1}\, c) + 4c^2\right] \tag{3.6}$$

which is minimized for

$$g = -\frac{2(2c^2 - \sqrt{\beta}\ c)}{(4c^2 - 4\sqrt{\beta_1}\ c + \beta_2 - 1)} = g_0 \text{ (say)} \tag{3.7}$$

$$= -\frac{2(2c^2 - \sqrt{\beta_1}\ c)}{\left[\beta_2 - \beta_1 - 1 + (2c - \sqrt{\beta_1})^2\right]}$$

Hence the resulting (minimum) mean square error of $\hat{\theta}_g$ is given by

$$\text{min.MSE}(\hat{\theta}_g) = \left(\frac{\theta^2 c^2}{n}\right) \frac{(\beta_2 - \beta_1 - 1)}{(4c^2 - 4\sqrt{\beta_1}\ c + \beta_2 - 1)} \tag{3.8}$$

4. *Funnel for estimators of* θ

From (2.2), (3.3) and (3.7), we have

$$\sum_{i=1}^{3} g_i = 1 \tag{4.1}$$

$$g_2 - g_3 = g_0 \tag{4.2}$$

From (4.1) and (4.2), we have three unknowns to be determined from only two equations. It is, therefore, not possible to find out unique values for the amount of chemicals g_i's (i = 1, 2, 3), we shall connect a filter with the funnel by imposing a linear restriction,

$$\sum_{i=1}^{3} g_i \ B(\hat{\theta}_i) = o \tag{4.3}$$

where $B(\hat{\theta}_i)$ is the bias in the i-th estimator of inverse of population mean. Now (4.1), (4.2) and (4.3) may be expressed as

$$\begin{pmatrix} o & 1 & -1 \\ 1 & 1 & 1 \\ B(\hat{\theta}_1) & B(\hat{\theta}_2) & B(\hat{\theta}_3) \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = \begin{pmatrix} g_o \\ 1 \\ o \end{pmatrix} \tag{4.4}$$

or $\quad A_{3x3}\ G_{3x1} = B_{3x1}$

The values of g_i's (i = 1, 2, 3) obtained by solving the system of equations (4.4) separate the bias precipitates from the suggested linear variety at (2.1) $|A| \neq o$. Thus we have the following theorem:

Theorem : The system of equations given by (4.4) will have a unique solution if

$$B(\hat{\theta}_1) \neq \frac{\{B(\hat{\theta}_2) + B(\hat{\theta}_3)\}}{2} \tag{4.5}$$

Proof : The proof of the theorem follows if we put $|A| \neq 0$.

5. *Bias separation of order* $O(n^{-1})$

We shall now outline the manner in which one can use the funnel connected with filter-paper to separate the bias precipitate of order $o(n^{-1})$ for the estimator $\hat{\theta}_g$ in (2.1). For the case under consideration, the biases of $\hat{\theta}_i$ (i = 1, 2, 3) to the first degree of approximation, are respectively given by

$$B(\hat{\theta}_1) = \theta c^2/n \tag{5.1}$$

$$B(\hat{\theta}_2) = \frac{\theta}{8n}\left[24c^2 - 8\sqrt{\beta_1}\,c - (\beta_2 - 1)\right] \tag{5.2}$$

$$B(\hat{\theta}_3) = \frac{3\theta}{8n}(\beta_2 - 1) \tag{5.3}$$

Expressions (5.1)-(5.3) clearly satisfy the condition (4.5). Using (5.1)-(5.3) in (4.4) and then solving we get the the unique solution as

$$g_1 = -\frac{1}{D}\left[g_o\{2(\beta_2 - 1) - 12c^2 + 4\sqrt{\beta_1}\,c\} - \{(\beta_2 - 1) + 12c^2 - 4\sqrt{\beta_1}\,c\}\right] \tag{5.4}$$

$$g_2 = -\frac{1}{2D}\left[g_o\{8c^2 - 3(\beta_2 - 1)\} + 8c^2\right] \tag{5.5}$$

$$g_3 = -\frac{1}{2D}\left[8c^2 + g_o\{16c^2 - 8\sqrt{\beta_1}\,c - (\beta_2 - 1)\}\right] \tag{5.6}$$

where $D = (4c^2 - 4\sqrt{\beta_1}\,c + \beta_2 - 1)$

Use of these g_i's (i = 1, 2, 3) filtrates the bias upto terms of order $o(n^{-1})$. Keeping in view the importance of the condition (4.5), the same process may be repeated by considering $\beta(\hat{\theta}_i)$ (i = 1, 2, 3) to the order $o(n^{-1})$ if the bias in $\hat{\theta}_g$ is to be reduced to the order $o(n^{-3})$ and so on.

6. *Normal Parent*

In case of normal population where $\beta_1 = o$ and $\beta_2 = 3$, the expression (5.2) to (5.6) respectively reduce to

$$B(\hat{\theta}_2) = \frac{\theta}{4n}(12c^2 - 1) \tag{6.1}$$

$$B(\hat{\theta}_3) = \frac{3\theta}{4n} \tag{6.2}$$

$$g_1 = \frac{(1 + 12c^{2)}}{(1 + 2c^2)^2} \tag{6.3}$$

$$g_2 = -\frac{(5c^2)}{(1 + 2c^2)^2} \tag{6.4}$$

$$g_3 = -\frac{(3c^2 - 4c^4)}{(1 + 2c^2)^2} \tag{6.5}$$

Thus the minimum variance of $\hat{\theta}_g$ is given by

$$\text{min.Var}(\hat{\theta}_g) = \frac{\theta^2}{n} \cdot \frac{c^2}{(1 + 2c^2)} \tag{6.6}$$

The MSE of $\hat{\theta}_1 = \frac{1}{\bar{y}}$, to the first degree of approximation, is given by

$$\text{MSE}(\hat{\theta}_{1)} = \frac{\theta^2 c^2}{n} \tag{6.7}$$

It follows from (6.6) and (6.7) that the relative efficiency (RE) of $\hat{\theta}_g$ with respect to conventional estimator $\hat{\theta}_1$ is given by

$$\text{RE}(\hat{\theta}_g, \hat{\theta}_1) = 1 + 2c^2 \tag{6.8}$$

which shows that proposed estimator $\hat{\theta}_g$ is more efficient than conventional estimator $\hat{\theta}_1 = -\frac{1}{\bar{y}}$.

Thus it is interesting to remark that the only prior knowledge of coefficient of variation is enough to use the proposed estimator $\hat{\theta}_g$ in case of normal population.

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4. The work must have been carried out in India and it should be an original contribution.
5. Four copies of the full paper along with copies of its abstract not exceeding 200 words should reach the Secretary of the Society not later than 28th October, preceding the Conference. Bio-data, including full name and address along with the date of birth (duly attested copy of certificate), research experience, list of publications should be appended to the complete paper.
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CONDOLENCE

The members of the Indian Society of Agricultural Statistics deeply mourn the sad demise of Prof. N. G. Ranga (Nidubrolu Gogineni Ranganayakulu) on 8 June, 1995. He was 94. Prof. Ranga was a founder member of the Society and one of the Vice Presidents from 1948 to 1953.

Prof. Ranga was a Gandhian, veteran parliamentarian, freedom fighter and above all a crusader of many a farmer's movement. But for a seven year break in Parliament between 1970 and 1977, Prof. Ranga was a member of either the Lok Sabha or the Rajya Sabha from the beginning. Prof. Ranga had the rare distinction of being awarded special honour for completing 50 years of parliamentary life.

As a member as well as Vice President of the Society he contributed significantly in promoting the cause of the Society. He delivered the prestigious "Dr. Rajendra Prasad Memorial Lecture" in 1981 during the 35th Annual Conference of the Society and the topic of his lecture was "Vista for continuous surveys to monitor progress in rural welfare planning". With the demise of Prof. N. G. Ranga the country in general and the Society in particular lost a renowned figure. The void left by him would be impossible to fill.